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Gauged $N = 2$ off-shell supergravity in five dimensions

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Abstract: We present some multiplets of $N = 2$ off-shell supergravity in five dimensions. One is the Super Yang-Mills multiplet, another one is the linear multiplet. The latter one is used to establish a general action formula from which we derive an action for the Super Yang-Mills multiplet. The Super Yang-Mills multiplet is used to construct the non-linear multiplet with gauged SU(2). This non-linear multiplet and the action formula for the Yang-Mills multiplet enable us to write down an SU(2) gauged supergravity which we finally truncate to arrive at gauged supergravity with gauge group SO(2).

Keywords: Field Theories in Higher Dimensions, Supergravity Models.

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Contents

1. Introduction

In a recent paper [1] we have presented an off-shell formulation of $N = 2$ supergravity in five spacetime dimensions. Our construction started from globally supersymmetric Yang-Mills theory from which we derived the multiplet of currents. By dualizing this multiplet we obtained the linearized transformation rules of the minimal multiplet which we subsequently completed to the full, non-linear ones. The field content of this multiplet is listed in table 1. The propagating fields are the vielbein e_m^a, the gravitino ψ_m and the graviphoton A_m. Besides these fields some auxilliary fields

Type	Field	SU(2)	dimension	components		
				bosonic		fermionic
vielbein	e_m^a	**1**	0	10		
graviphoton	A_m	**1**	3/2	4		
gravitino	ψ_m	**2**	2			32
isotriplet	$\vec{t}$	**3**	5/2	3		
antisymmetric tensor	v_{ab}	**1**	5/2	10		
SU(2) gauge field	$\vec{V}_m$	**3**	5/2	12		
spinor	λ	**2**	3			8
scalar	C	**1**	7/2	1		
				40	+	40

Table 1: Field content of the minimal multiplet in $D = 5$.

are needed which ensure the matching of bosonic and fermionic degrees of freedom off the mass shell. As can be seen from the table, the off-shell extension requires all in all $40 + 40$ components. Since at least $48 + 48$ components are needed in order to construct a supergravity lagrangian, $8 + 8$ components are missing. These $8 + 8$ components were added to the theory by introducing the non-linear multiplet. Furthermore, using this multiplet, we were able to break the SU(2) automorphisms which were gauged by the auxilliary field $\vec{V}_m$ to a global SU(2), leading to an off-shell lagrangian of supergravity.

In this note we continue the construction of locally supersymmetric off-shell theories in $D = 5$. We use the minimal formulation throughout, i.e. with unbroken SU(2) and $40 + 40$ components. At the end, the non-linear multiplet is introduced leading to the desired lagrangians. Our final result will be gauged supergravity, i.e. AdS supergravity (for a recent review on supersymmetry in AdS space, see [2]). In principle there are two possible versions of gauged supergravities:

The first one consists in gauging the complete automorphism group by introducing an additional Super Yang-Mills multiplet with gauge group $\mathrm{SU}(2)'$. The resulting $\mathrm{SU}(2) \times \mathrm{SU}(2)'$ is then broken to a residual SU(2) which gauges the complete automorphisms, where in contradistinction to the minimal case, the gauge field is now propagating. We will not present the explicit lagrangian but explain in some detail its construction.

The second formulation of gauged supergravity uses the graviphoton A_m belonging to the supergravity multiplet to gauge an SO(2) subgroup of the automorphism group. The resulting theory contains a cosmological constant. The lagrangian will be presented in section 4.

The plan for this note is as follows. In the next section we find the transformation laws of the locally supersymmetric Yang-Mills multiplet. In section 3 the linear multiplet is presented. It allows the construction of a general action formula which is used to find an action for the Super Yang-Mills multiplet. In section 4 we move on to the main topic of this work, the construction of gauged supergravity. This requires the inclusion of a non-abelian gauge symmetry in the non-linear multiplet is presented there. Finally, we give the lagrangian for off-shell supergravity in Anti de Sitter space.

2. The Super Yang-Mills multiplet

The field content of the Super Yang-Mills multiplet is given in table 2. We found the locally supersymmetric transformation laws starting from the rigid ones which are given in [12] by imposing the commutator algebra of the minimal multiplet, i.e. the multiplet containing $40 + 40$ components as constructed in our recent work [1]. One finds that this algebra has to be modified. In addition to the terms given there, a field

Type	Field	SU(2)	dimension	components	
				bosonic	fermionic
gauge field	B_m^A	**1**	3/2	$5-1$	
scalar	M^A	**1**	3/2	1	
gaugino	Ω^A	**2**	2		8
isotriplet	$\vec{X}^A$	**3**	5/2	3	
				8 +	8

Table 2: Field content of the Super Yang-Mills multiplet in $D=5$. The index A is a gauge group index.

dependent gauge transformation appears in the commutator of two supersymmetry transformations:

$$[\delta_Q(\eta),\delta_Q(\varepsilon)] = \cdots + \delta_{\text{gauge}}(ig\bar{\varepsilon}\gamma^m\eta B_m^A - ig\bar{\varepsilon}\eta M^A)\,. \tag{2.1}$$

The dots represent the field dependent transformations of [1], $\delta_{\text{gauge}}(\alpha)$ is a gauge transformation with parameter α and g is the gauge coupling constant which has (mass-) dimension $-1/2$ in five dimensions.

The tranformation laws we find are

$$\begin{aligned}
\delta B_m^A &= i\bar{\varepsilon}\gamma_m\Omega^A - i\bar{\varepsilon}\psi_m M^A\,,\\
\delta M^A &= i\bar{\varepsilon}\Omega^A\,,\\
\delta\Omega^A &= \frac{1}{4}\gamma^{ab}\varepsilon\widehat{G}_{ab}^A - \frac{1}{2}\gamma^a\varepsilon\widehat{D}_a M^A + i\vec{X}^A\vec{\tau}\varepsilon - \frac{1}{2\sqrt{3}}\gamma^{ab}\varepsilon\widehat{F}_{ab}M^A\,,\\
\delta\vec{X}^A &= -\frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^a\widehat{D}_a\Omega^A - \frac{g}{2}f_{BC}{}^A\bar{\varepsilon}\vec{\tau}\Omega^B M^C + \frac{1}{4}\bar{\varepsilon}\vec{\tau}\gamma^{ab}\Omega^A v_{ab} + 4i\bar{\varepsilon}\Omega^A\vec{t} - i\bar{\varepsilon}\vec{\tau}\tilde{\tau}\Omega^A\tilde{t}\,,
\end{aligned} \tag{2.2}$$

where $f_{AB}{}^C$ are the structure constants of the Lie algebra

$$[T_A, T_B] = if_{AB}{}^C T_C\,.$$

Our notations and conventions are the same as in our previous work [1], except for the gravitational coupling κ which is now set to one. For the convenience of the reader we list the explicit expressions for the supercovariant objects as they appear in the transformation laws (2.2):

$$\begin{aligned}
\widehat{D}_m M^A &= \partial_m M^A + gf_{BC}{}^A B_m^B M^C + i\bar{\Omega}^A\psi_m\,,\\
\widehat{G}_{mn}^A &= \partial_m B_n^A - \partial_n B_m^A + gf_{BC}{}^A B_m^B B_n^C - 2i\bar{\psi}_{[m}\gamma_{n]}\Omega^A + i\bar{\psi}_m\psi_n M^A\,,\\
\widehat{D}_m\Omega^A &= \partial_m\Omega^A + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\Omega^A - \frac{i}{2}\vec{V}_m\vec{\tau}\Omega^A + gf_{BC}{}^A B_m^B\Omega^C - \\
&\quad - \frac{1}{4}\gamma^{ab}\psi_m\widehat{G}_{ab}^A + \frac{1}{2}\gamma^a\psi_m\widehat{D}_a M^A - i\vec{\tau}\psi_m\vec{X}^A + \frac{1}{2\sqrt{3}}\gamma^{ab}\psi_m\widehat{F}_{ab}M^A.
\end{aligned}$$

The next step would be to construct an action for this multiplet. However, as will turn out and as expected from the four-dimensional case [4], this action is rather

Type	Field	SU(2)	components	
			bosonic	fermionic
isotriplet	$\vec{Y}$	**3**	3	
fermion	ρ	**2**		8
scalar	N	**1**	1	
vector	W_a	**1**	$5-1$	
			8 +	8

Table 3: Field content of the linear multiplet in $D = 5$. The subtracted degree of freedom of the vectorfield is due to the constraint (3.1).

complicated. To circumvent its direct construction, it is useful to construct first the linear multiplet, for which an action is easily found. Since a certain combination of fields belonging to the Super Yang-Mills multiplet forms a linear multiplet, this formula can then be used to give an action for the Super Yang-Mills multiplet, as will be discussed at the end of the next section.

3. The linear multiplet

The linear multiplet is a useful device for the construction of invariant actions [5]. Since the highest component of this multiplet is a scalar N which transforms in the globally supersymmetric case into a total derivative, an invariant is given by

$$\sim \int d^5x \; N \, .$$

This relation is easily extended to the locally supersymmetric case. It will be shown below, following the construction in the four-dimensional case [4], that a certain combination of components of the minimal multiplet as constructed in [1] forms a linear multiplet, SO that knowing an action for the linear multiplet, an action for the minimal multiplet is known, too. Furthermore, the Super Yang-Mills multiplet can be embedded in a similar manner in the linear multiplet which will simplify the construction of an action for this multiplet considerably.

The complete field content of the linear multiplet is given in table 3. It contains a vector which is conserved in the rigid supersymmetric limit, i.e. $\partial \cdot W = 0$. Of course, this constraint will be modified in the locally supersymmetric case; its actual form is given in eq. (3.1).

In the globally supersymmetric limit, the linear multiplet is dual to the Maxwell multiplet, i.e. by requiring invariance of the coupling term[1]

$$\sim \int d^5x \left(B_a W^a + MN + i\bar{\Omega}\rho - \vec{X}\vec{Y} \right)$$

[1]From this point of view the conservation of the vector W_a is a direct consequence of the fact that B_a is a gauge field.

we deduced the rigid transformation laws of the linear multiplet which were then completed to the locally supersymmetric ones by enforcing the gauge algebra (2.1).

In addition, the fields may transform in a real representation of a non-abelian gauge group. If such a gauge symmetry is included, the fields tranform like

$$\delta N = i\varepsilon^A T_A N\,,$$

where the T^A generate this representation and ε^A is the transformation parameter. The supersymmetry transformation laws we find are

$$\begin{aligned}
\delta\vec{Y} &= \bar{\varepsilon}\vec{\tau}\rho\,,\\
\delta\rho &= \gamma^a\varepsilon W_a + \varepsilon N - \frac{i}{2}\gamma^a\vec{\tau}\varepsilon\widehat{D}_a\vec{Y} + i\gamma^{ab}\vec{\tau}\varepsilon v_{ab}\vec{Y} + \\
&\quad + 6i\vec{\tau}\varepsilon(\vec{t}\times\vec{Y}) - \frac{i}{2\sqrt{3}}\gamma^{ab}\vec{\tau}\varepsilon\widehat{F}_{ab}\vec{Y} + \frac{g}{2}\vec{\tau}\varepsilon M\vec{Y}\,,\\
\delta W^a &= \frac{i}{2}\bar{\varepsilon}\gamma^{ab}\widehat{D}_b\rho + \frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^{abc}\widehat{\mathcal{R}}_{bc}\vec{Y} - \frac{i}{2}\bar{\varepsilon}\gamma^{abc}\rho v_{bc} + \frac{3i}{2}\bar{\varepsilon}\gamma_b\rho v^{ab} + \\
&\quad + 4\bar{\varepsilon}\vec{\tau}\gamma^a\rho\vec{t} + \frac{i}{2\sqrt{3}}\bar{\varepsilon}\gamma^{abc}\rho\widehat{F}_{bc} - \frac{i}{2\sqrt{3}}\bar{\varepsilon}\gamma_b\rho\widehat{F}^{ab} + \frac{ig}{2}\bar{\varepsilon}\vec{\tau}\gamma^a\Omega\vec{Y} + \frac{g}{2}\bar{\varepsilon}\gamma^a M\rho\,,\\
\delta N &= -\frac{i}{2}\bar{\varepsilon}\gamma^a\widehat{D}_a\rho - \frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^{ab}\widehat{\mathcal{R}}_{ab}\vec{Y} - 8\bar{\varepsilon}\vec{\tau}\lambda\vec{Y} - \frac{3i}{4}\bar{\varepsilon}\gamma^{ab}\rho v_{ab} - 3\bar{\varepsilon}\vec{\tau}\rho\vec{t} - \frac{ig}{2}\bar{\varepsilon}\vec{\tau}\Omega\vec{Y}\,.
\end{aligned}$$

Here we use Lie-algebra valued expressions for the fields belonging to the Super Yang-Mills multiplet, e.g. $\Omega = \Omega^A T_A$. Explicitly, the supercovariant derivatives appearing in the transformation laws are

$$\begin{aligned}
\widehat{D}_m\vec{Y} &= \partial_m\vec{Y} + \vec{V}_m\times\vec{Y} - igB_m\vec{Y} - \bar{\rho}\vec{\tau}\psi_m\,,\\
\widehat{D}_m\rho &= \partial_m\rho + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\rho - \frac{i}{2}\vec{V}_m\vec{\tau}\rho - igB_m\rho - \gamma^a\psi_m W_a - \psi_m N + \frac{i}{2}\gamma^a\vec{\tau}\psi_m\widehat{D}_a\vec{Y} - \\
&\quad - i\gamma^{ab}\vec{\tau}\psi_m v_{ab}\vec{Y} - 6i\vec{\tau}\psi_m(\vec{t}\times\vec{Y}) + \frac{i}{2\sqrt{3}}\gamma^{ab}\vec{\tau}\psi_m\widehat{F}_{ab}\vec{Y} - \frac{g}{2}\vec{\tau}\psi_m M\vec{Y}\,,\\
\widehat{D}_m N &= \partial_m N - igB_m N + \frac{i}{2}\bar{\psi}_m\gamma^a\widehat{D}_a\rho + \frac{1}{2}\bar{\psi}_m\vec{\tau}\gamma^{ab}\widehat{\mathcal{R}}_{ab}\vec{Y} + \\
&\quad + \bar{\psi}_m\vec{\tau}\lambda\vec{Y} + \frac{3i}{4}\bar{\psi}_m\gamma^{ab}\rho v_{ab} + 3\bar{\psi}_m\vec{\tau}\rho\vec{t} + \frac{ig}{2}\bar{\psi}_m\vec{\tau}\Omega\vec{Y}
\end{aligned}$$

and the constraint which was mentioned above and which is required for closure of the algebra on N and W_a is

$$\widehat{D}_a W^a + \frac{i}{2}\bar{\rho}\gamma^{ab}\widehat{\mathcal{R}}_{ab} - 8igM\vec{Y}\vec{t} + ig\vec{X}\vec{Y} + g\bar{\Omega}\rho - igMN = 0\,. \tag{3.1}$$

The supercovariant derivative on W^a which appears in this expression is defined by

$$\begin{aligned}\widehat{D}_m W^a &= \partial_m W^a + \widehat{\omega}_m{}^{ab} W_b - igB_m W^a - \frac{i}{2}\bar{\psi}_m\gamma^{ab}\widehat{D}_b\rho - \frac{1}{2}\bar{\psi}_m\gamma^{abc}\vec{\tau}\widehat{\mathcal{R}}_{bc}\vec{Y} + \\ &\quad + \frac{i}{2}\bar{\psi}_m\gamma^{abc}\rho v_{bc} - \frac{3i}{2}\bar{\psi}_m\gamma_b\rho v^{ab} - 4\bar{\psi}_m\vec{\tau}\gamma^a\rho\vec{t} - \frac{i}{2\sqrt{3}}\bar{\psi}_m\gamma^{abc}\rho\widehat{F}_{bc} + \\ &\quad + \frac{i}{2\sqrt{3}}\bar{\psi}_m\gamma_b\rho\widehat{F}^{ab} - \frac{ig}{2}\bar{\psi}_m\vec{\tau}\gamma^a\Omega\vec{Y} - \frac{g}{2}\bar{\psi}_m\gamma^a M\rho\,.\end{aligned}$$

We now give an invariant for the case of vanishing gauge group: $S = \int d^5x\ e\mathrm{L}$, with a lagrangian

$$\begin{aligned}\mathrm{L} = -4N + 2i\bar{\psi}_a\gamma^a\rho - 32\vec{t}\vec{Y} + \bar{\psi}_a\vec{\tau}\gamma^{ab}\psi_b\vec{Y} - \\ - \frac{2}{\sqrt{3}}(4W^a + 2i\bar{\rho}\gamma^{ab}\psi_b + \bar{\psi}_b\vec{\tau}\gamma^{abc}\psi_c\vec{Y})A_a\end{aligned} \tag{3.2}$$

is invariant under supersymmetry and U(1) gauge transformations. To check the latter symmetry the constraint (3.1) has to be used. Note the formal similarity of this action with the four-dimensional result [4].

As stated above, a certain combination of fields belonging to the minimal multiplet forms a linear multiplet. The exact correspondence is

$$(\vec{Y}, \rho, N, W^a) = \left(\vec{t}, \lambda, C, \frac{1}{4}\widehat{D}_b v^{ab} + \frac{1}{48}\varepsilon^{abcde}\widehat{F}_{bc}\widehat{F}_{de}\right)\,.$$

Using this relation in (3.2) leads to the action for the minimal multiplet given in [1].

As for the minimal multiplet it is also possible for the Super Yang-Mills multiplet of the preceding section to find certain combinations of fields which form the components of a linear multiplet. Since we know an action formula for this multiplet, eq. (3.2), we can find in this way an action for the Super Yang-Mills multiplet. Because the exact correspondence is rather complicated, we give only the lowest-dimensional field of the linear multiplet:

$$\begin{aligned}(\vec{Y}^{AB}, \rho^{AB}, N^{AB}, W_a^{AB}) = \Bigg(\frac{1}{2}\bigg(M^A\vec{X}^B + M^B\vec{X}^A + \\ + \frac{1}{2}\bar{\Omega}^A\vec{\tau}\Omega^B - 4M^AM^B\vec{t}\bigg), \ldots\Bigg)\,.\end{aligned} \tag{3.3}$$

The remaining components of this linear multiplet can then be computed by repeated supersymmetry variation of this expression. Note again the similarity of this result with the result of Breitenlohner and Sohnius [4].

The next step is to project out a gauge singlet by considering the contraction of the indices A and B in (3.3). We write this as a trace. The lagrangian density for the Super Yang-Mills multiplet is then:

$$
\begin{aligned}
\mathrm{L}_{YM} = \operatorname{tr}\Bigg(& -\frac{1}{4}\widehat{G}_{ab}\widehat{G}^{ab} + \frac{1}{2}\widehat{D}_a M\widehat{D}^a M + 2\vec{X}\vec{X} + i\bar{\Omega}\gamma^m\widehat{D}_m\Omega + 64MM\vec{t}\vec{t} - \\
& - 16M\vec{X}\vec{t} + 2\left(\widehat{G}_{ab} - \frac{1}{\sqrt{3}}M\widehat{F}_{ab}\right)\left(v^{ab} + \frac{1}{2\sqrt{3}}\widehat{F}^{ab}\right)M + 8CM^2 + \\
& + \frac{1}{4\sqrt{3}}\varepsilon^{abcde}A_a G_{bc}G_{de} - 16i\bar{\lambda}\Omega M - \frac{i}{2}\bar{\Omega}\gamma^{ab}\Omega v_{ab} - 2\bar{\Omega}\vec{\tau}\Omega\vec{t} + \\
& + g[\bar{\Omega},\Omega]M - \bar{\psi}_a\vec{\tau}\gamma^a\Omega\vec{X} + 8\bar{\Omega}\vec{\tau}\gamma^a\psi_a\vec{t}M - \frac{i}{2}\bar{\Omega}\gamma^a\gamma^b\psi_b\widehat{D}_a M - \\
& - \frac{i}{4}\bar{\psi}_a\gamma^a\gamma^{bc}\Omega\widehat{G}_{bc} + 4i\bar{\Omega}\gamma_a\psi_b v^{ab}M + \frac{i}{2\sqrt{3}}\bar{\psi}_a\gamma^a\gamma^{bc}\Omega\widehat{F}_{bc}M + \\
& + \frac{i}{4}\bar{\psi}_m\gamma^{mnab}\psi_n\widehat{G}_{ab}M - 4i\bar{\psi}_a\gamma^a\lambda M^2 - 2\bar{\psi}_a\vec{\tau}\gamma^{ab}\psi_b\vec{t}M^2 - \\
& - \frac{i}{4\sqrt{3}}\bar{\psi}_m\gamma^{mnab}\psi_n\widehat{F}_{ab}M^2 - iv^{ab}\bar{\psi}_a\psi_b M^2 + \frac{1}{4}\bar{\psi}_a\vec{\tau}\gamma^{ab}\psi_b\bar{\Omega}\vec{\tau}\Omega + \\
& + \frac{1}{4}\bar{\Omega}\gamma^{ab}\Omega\bar{\psi}_a\psi_b - \frac{1}{2}\bar{\psi}_n\gamma^a\psi_m\bar{\Omega}\gamma^{mn}\psi_a M + \bar{\psi}_p\gamma^p\psi_m\bar{\Omega}\gamma^{mn}\psi_n M - \\
& - \bar{\Omega}\gamma^{mn}\psi_n\bar{\Omega}\psi_m + \frac{1}{2}\bar{\Omega}\gamma^{mnp}\psi_n\bar{\psi}_m\psi_p M + \frac{1}{8}\bar{\psi}_m\gamma^{mnpq}\psi_n\bar{\psi}_p\psi_q M^2\Bigg)\,. \qquad (3.4)
\end{aligned}
$$

Note the appearance of a term linear in C (the last term in the second line), as expected from the counting of degrees of freedom. As in the pure supergravity case, this field has to be replaced using the non-linear multiplet.

4. Gauged supergravity

As discussed in the introduction, there are two possibilities for gauged supergravity. First, one can gauge the complete automorphisms by introducing in addition to the supergravity multiplet an SU(2) vector multiplet. The second posssibility is to gauge an SO(2) subgroup of the SU(2), where the gauge field of the SO(2) is the graviphoton, which belongs to the supergravity multiplet. We will discuss the first possibility only roughly, following the pioneering work of de Wit, [6]. For the second possibility, i.e. AdS supergravity, we will present detailed formulas. Our strategy in this case follows closely the work of de Wit et al. on four-dimensional conformal supergravity, [7].

To gauge the automorphism group of the supersymmetry algebra we have to return to the non-linear multiplet which has been constructed in [1]. Its components are listed in table 4. A constraint of the form $\partial_a V^a + \cdots = 0$ was required to render this multiplet supersymmetric. The index α of ${\phi^i}_\alpha$ transformed under global SU(2)$'$ transformations. The first step towards gauged supergravity is to gauge this SU(2)$'$ by introducing a Super Yang-Mills multiplet with components

$$(\vec{B}_m, \vec{M}, \vec{\Omega}, \vec{\vec{X}})\,, \qquad (4.1)$$

Type	Field	$SU(2)\times SU(2)'$	dimension	components	
				bosonic	fermionic
scalar	${\phi^i}_\alpha$	$(\mathbf{2},\mathbf{2})$	0	3	
spinor	χ	$(\mathbf{2},\mathbf{1})$	2		8
scalar	φ	$(\mathbf{1},\mathbf{1})$	5/2	1	
vector	V_a	$(\mathbf{1},\mathbf{1})$	5/2	$5-1$	
				8 +	8

Table 4: Field content of the non-linear multiplet. The subtracted degree of freedom of V_a is due to the constraint (4.3).

where the arrows denote the **3** of $SU(2)'$ and the tilde on $\vec{\tilde{X}}$ denotes the **3** of the automorphism group $SU(2)$. To gauge the $SU(2)'$, the derivatives acting on ${\phi^i}_\alpha$ which appear in various transformation laws of fields belonging to the non-linear multiplet have to be replaced by gauge covariant derivatives:

$$\partial_m {\phi^i}_\alpha \longrightarrow \partial_m {\phi^i}_\alpha + \frac{ig}{2}{\phi^i}_\beta {(\vec{\tau})^\beta}_\alpha \vec{B}_m \,. \tag{4.2}$$

However, imposing the commutator algebra with the modification due to the extra $SU(2)'$ given in eq. (2.1) on the non-linear multiplet requires further modifications of the transformation laws. These are (besides the implicit replacements of the derivatives by covariant derivatives, cf. (4.2), and various supercovariantizations):

$$\begin{aligned}
\Delta\chi^i &= \frac{-ig}{4}{\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j \varepsilon^j \vec{M}\,,\\
\Delta\varphi &= \frac{g}{2}\bar{\varepsilon}_i \chi^j {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j \vec{M} + \frac{g}{2}\bar{\varepsilon}_i \vec{\Omega}^j {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j\,,\\
\Delta V_a &= \frac{g}{2}\bar{\varepsilon}_i \gamma_a \chi^j {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j \vec{M} + \frac{g}{2}\bar{\varepsilon}_i \gamma_a \vec{\Omega}^j {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j\,,
\end{aligned}$$

so that $\delta^{\text{new}} = \delta^{\text{old}} + \Delta$. Furthermore, the constraint of the non-linear multiplet as given in [1] is modified. Besides the implicit modifications due to covariantizations and supercovariantizations there appear also extra terms:

$$\begin{aligned}
C + \cdots - \frac{g}{8}\vec{\tilde{X}}{(\tilde{\tau})^j}_i {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j - \frac{g}{4}\bar{\chi}_i(\chi^j \vec{M} + 2\vec{\Omega}^j){\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j +&\\
+\frac{g}{4}{(\tilde{\tau})^j}_i {\phi^i}_\alpha {(\vec{\tau})^\alpha}_\beta {\phi^\beta}_j \vec{M}\tilde{t} + \frac{g^2}{16}\vec{M}^2 = 0\,.&
\end{aligned} \tag{4.3}$$

The dots represent again the terms already present in [1] and for clarity we have given the C-term.

To gauge the complete $SU(2)$ automorphism group with a propagating field one should proceed as follows. First, one has to plug in the modified constraint (4.3) in the Super Yang-Mills lagrangian (3.4). Next, the $SU(2)\times SU(2)'$ is broken to a residual $SU(2)$ by choosing

$${\phi^i}_\alpha = \delta^i_\alpha\,. \tag{4.4}$$

To maintain this gauge, a field dependent SU(2) transformation with parameter

$$\vec{\xi} = 2\bar{\varepsilon}\vec{\tau}\chi + \vec{\eta}$$

has to be added to the transformation laws of the fields. Here $\vec{\eta}$ is the transformation parameter of the $\mathrm{SU}(2)'$. As a consequence, there are now two fields, $\vec{B}_m$ and $\vec{V}_m$, transforming as gauge fields of the residual SU(2). Further, there is a derivative term $\sim \partial\varepsilon$ in the supersymmetry transformation law of $\vec{V}_m$. All this can be remedied by redefining

$$\vec{B}'_m = \vec{V}_m - g\vec{B}_m + 2\bar{\psi}_m\vec{\tau}\chi \tag{4.5}$$

$\vec{B}'_m$ is now inert under gauge transformations and of auxilliary dimension.

To gauge an SO(2) subgroup of the SU(2) automorphism group one could proceed by setting

$$\vec{\Omega} = \vec{\tilde{X}} = 0\,, \qquad \vec{M} = \frac{1}{\sqrt{2}}(0,1,0)^T\,, \qquad \vec{B}_m = \sqrt{\frac{2}{3}}(0,A_m,0)^T\,. \tag{4.6}$$

Inspection of the supersymmetry transformation laws for the Super Yang-Mills multiplet (2.2) shows that this choice is supersymmetric. Furthermore, one should perform the following redefinitions which lead to canonical kinetic terms:

$$\lambda' = \lambda + \frac{1}{4}\gamma^{ab}\widehat{\mathcal{R}}_{ab}\,, \qquad v'_{ab} = v_{ab} - \frac{1}{2\sqrt{3}}\widehat{F}_{ab}\,. \tag{4.7}$$

However, a shortcut of this procedure is to start by imposing (4.6) and the redefinitions (4.7) first. After having replaced the C field in the lagrangian by the constraint (4.3) and having chosen the gauge (4.4), the original $\mathrm{SU}(2)\times\mathrm{SU}(2)'$ is broken to SO(2). To stay in the gauge (4.4), an SU(2) transformation with parameter

$$\vec{\xi} = 2\bar{\varepsilon}\vec{\tau}\chi + (0,\eta^2,0)^T$$

has to be addded to the supersymmetry transformation laws. Again, as in the case with gauged SU(2) discussed above, there are now two fields which transform as gauge fields for the SO(2), namely V^2_m and A_m. To get rid of this, we define a new vectorfield

$$\vec{V}'_m = \vec{V}_m - g\sqrt{\frac{2}{3}}(0,A_m,0)^T - 2\bar{\psi}_m\vec{\tau}\chi$$

which is inert under the SO(2). After replacing

$$\vec{t} \longrightarrow \vec{t} + \frac{g}{12\sqrt{2}}(0,1,0)^T$$

the lagrangian becomes

$$\mathrm{L} = \mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_3 + \frac{g'^2}{\kappa^4} - \sqrt{3}\frac{g'}{\kappa}\bar{\chi}\tau^2\chi - \frac{\sqrt{3}g'}{4\kappa}\bar{\psi}_a\tau^2\gamma^{ab}\psi_b\,,$$

where we have restored the gravitational coupling constant κ. Here $\mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_3$ are the lagrangians of the ungauged theory given in [1], with the derivatives acting on the spinor doublets ψ_m and χ replaced by gauge covariant derivatives,

$$\partial_m \longrightarrow \partial_m - ig' A_m \tau^2$$

and we have defined a new gauge coupling constant $g' = g/\sqrt{6}$. On-shell only

$$\mathrm{L} = \mathrm{L}_1 + \frac{g'^2}{\kappa^4} - \frac{\sqrt{3}g'}{4\kappa}\bar{\psi}_a \tau^2 \gamma^{ab} \psi_b$$

remains. This on-shell lagrangian agrees up to conventions with a special case of the more general theory discussed in [8]. It appeared first in the literature in [9]. Thus we suceeded in extending these works to off-shell formulations.

Let us comment on some prospects which are offered by the present work. Using our off-shell formulation it is rather simple to couple various matter multiplets to the theory. In particular, it would be of great interest to derive the most general coupling of gauged supergravity to several Maxwell multiplets from off-shell supergravity. This should shed some light on the origin of the appearing geometric structures [8], as it did in four dimensions [10].

Another quite similar application is the supersymmetrization of brane world scenarios à la Randall-Sundrum [11]. The procedure is a straightforward generalization of the techniques developed in [12]: One considers gauged supergravity on an orbifold $S^1/\mathbb{Z}_2$. The parity assignments of the bulk fields under the $\mathbb{Z}_2$ are dictated by supersymmetry. One finds that the even parity fields of the minimal multiplet form a non-minimal supergravity multiplet of $N = 1$, $D = 4$ supergravity at the fixpoints of the orbifold. This multiplet and the complete multiplet calculus where presented in [13]. The strategy should then be clear: One constructs lagrangians $\mathrm{L}_{\mathrm{matter}}$ for various four-dimensional matter multiplets using the tensor calculus of Sohnius and West and considers actions which look schematically like

$$S = \int d^5xe \left(\mathrm{L}_{\mathrm{gauged}} + \delta(x^5)\mathrm{L}_{\mathrm{matter}} + \delta(x^5 - \ell)\mathrm{L}'_{\mathrm{matter}}\right) .$$

Elimination of the auxiliary fields then leads to the desired results. We stress that this procedure requires inherently off-shell formulations of the theories in question. The details of this construction will be presented in a future publication [14].

Note added. After this work was completed, a paper [15] appeared, which has some overlap with the present work.

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